A WBL-CONSISTENT JWL EQUATION OF STATE FOR THE HMX-BASED EXPLOSIVE EDC37 FROM CYLINDER TESTS

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Historically, high explosive (HE) equations of state (EoS) derived from simple cylinder tests have proved inadequate when applied to scenarios with different geometries or confinements. The Witham-Bdzil-Lambourn (WBL) model for the propagation of a steady detonation wave, which accurately models its wave front characteristics, proving itself better than simple Huygens construction, has been successfully applied to the HMX-based explosive EDC37. The HE EoS therefore had to be recalculated to be consistent with this new propagation method, so the cylinder test was revived and re-applied to EDC37. The resulting EoS parameters were tested on simulations of a number of geometries; the results were extremely good. For the first time, a cylinder test-derived EoS has successfully modelled varied scenarios without recourse to arbitrary adjustment of the HE material parameters at run-time.

RATIONALE

The cylinder test is a conceptually simple experiment whereby a cylinder of HE encased in a close fitting metal tube is detonated at one end. The subsequent motion of the metal, accelerated by the expanding detonation products, is measured using a range of diagnostic techniques. The original test was conceived as a reproducible means of assessing the comparative performance of new explosive compositions, but refinements to the experimental techniques, yielding more accurate data, led to its use in determining explosion products EoS by comparison with 2D calculations.

EoS deduced in this way have typically failed to be of use in modelling weapon scenarios. This is a result of the fact that the plane waves predicted in the hydrocodes never really existed, as perturbations from the walls of the cylinder gave a drag to the edges of the detonation wave, curving it. The plane wave assumption ultimately led to higher detonation pressure being deduced because the experimentally observed jump off velocity is higher with a curved detonation wave than with a plane wave assumed in the code. This in turn led to a different set of values for the remaining parameters (needed to match the artificially high ‘CJ’ pressure deduced) to those that would otherwise have been obtained had the wave been modelled correctly.

Similarly, a CJ approach to detonation propagation ignores the real structure of the detonation wave, which terminates at the sonic point where CJ conditions prevail. This simplification has implications on the energy release behind the detonation wave when travelling at a speed less than the CJ velocity. The speed reduction is physically brought about by a lack of compression in the reaction zone, resulting in a lower peak pressure and higher product entropy. Manipulating the burn field in the code to reduce the local detonation speed results in inflated compression with reduced product entropy and over estimation of the peak pressure.

With the advent of the WBL detonation wave propagation method, the capability to accurately predict the shape and speed of a diverging detonation wave became a reality.

To therefore ensure accuracy in the HE products EoS, the cylinder test was fielded to acquire data on EDC37 with respect to the WBL model.

WBL DETONATION PROPAGATION MODEL

The WBL model is a first order correction to Huygens propagation, accounting for the changing speed of a divergent detonation wave (through a speed-curvature relation) and the effect inert boundaries have on the detonation wave. This is all without recourse to resolving the reaction zone explicitly.

Both a speed-curvature relation and boundary angle values have been formulated for EDC37.

EXPERIMENTAL FORMULATION

Each test was a 300mm long, 25.4mm diameter cylinder of HE, encased in a 2.6mm thick C101 copper sleeve. The tolerance on the fit between the two components was controlled to within 25μm. In early experiments, a plane wave lens was used to initiate the cylinder, but with the emergence of WBL it became clear that irrespective of the initiation pattern, a steady state, curved detonation front would propagate along the length of the experiment. A single EBW detonator was therefore fielded in these more recent rounds removing all complexities of plane wave initiation.

The expansion of the sleeve due to the detonation products is captured with arrays of probes angled to record near-simultaneous events to avoid causal noise. These inclined probe arrays can be seen in Figure 1. Each probe array consists of a line of copper conductors on a printed circuit board. These probes give a measure of the full motion of the cylinder wall from zero to 20mm expansion, by which time the sleeve is typically breaking up. At this stage, the products have expanded to about 7 times the initial volume. The probes are charged prior to firing, and the cylinder earthed. When the wall of the cylinder makes contact with a probe current flows, and a signal is recorded.

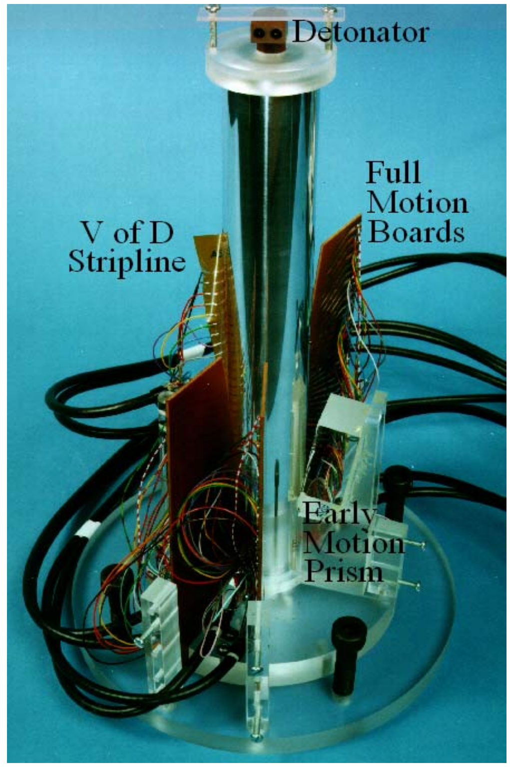

FIGURE 1: A TYPICAL CYLINDER TEST

The detonation speed of the explosive is measured with another array of probes, this time adhered to the outside of the cylinder wall. They are spaced off the wall by a thin melamine sheet and, like the full motion probes, are charged before firing.

The early stages of wall motion are dominated by the shock reverberations in the metal wall. The detonation wave transmits a shock into the metal which, on reaching the free surface, accelerates it and reflects a rarefaction back into the material. In turn, this is reflected forwards from the products/metal interface to give a further acceleration. This discontinuous behaviour is primarily dictated by the detonation pressure rather than the product expansion and its measurement provides data to determine this starting point in the EoS. As the discrete spacing of probes is incapable of resolving this behaviour, an optical total internal reflection method was chosen. A glass block (prism) is placed with one edge in contact with the cylinder and tilted to face the approaching metal (Figure 1). The angle of inclination is chosen to approximate that expected for the metal during the period of measurement to ensure the phase speed of contact with the glass is much higher than that of any shocks present. An external source provides light that is totally internally reflected from the base of the prism. With the slit of a rotating mirror streak camera aligned across the centre of the prism, corresponding to a radial plane, this reflected light is continuously recorded.

When the cylinder comes into contact with the base of the prism, the total internal reflection ceases and a temporal recording of the impact profile of the cylinder is obtained (Figure 2).

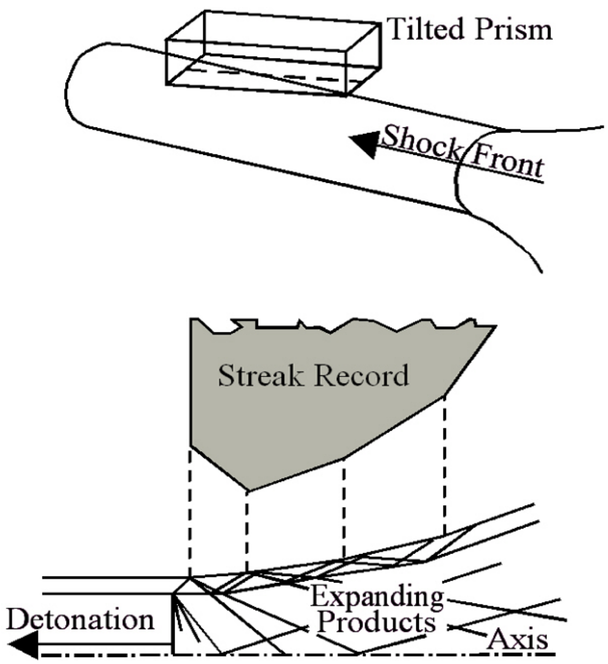

FIGURE 2: EARLY MOTION

A measurement of the detonation wave shape emerging from the end of the cylinder was made during one of the cylinder tests, supplementing the support experiments designed to further validate the WBL prediction. In the absence of a second synchronisable streak camera, the early motion diagnostic was omitted from this round. A small gap was present at the end of the explosive so trapped air would flash when the shock wave compressed it.

To ensure reproducibility, three full-length copper cylinder tests were fired in this series, along with a number of WBL support rounds.

CYLINDER MOTION BY STEADY ANALYSIS

The results from both early and full motion are a recording of the arrival of the cylinder wall along inclined planes. These essentially define its shape rather than giving any information about the expansion history. Since each measurement occupies a unique spatial position, it is very difficult to compare results of arrays on the same or different experiments. Such a comparison is feasible with hydrocode calculations, provided that the free surface arrival time at the unique spatial points can be obtained. Accuracy of this is limited by the experimental uncertainties in the absolute positions of the diagnostics, round to round variability in detonator functioning time, any asymmetry in the detonation wave and small differences in cylinder wall thicknesses.

A consequence of assuming steady conditions is self similar flow; all points along the cylinder follow the same trajectory with only the start time being different. The difference in start time is simply the time it takes the shock in the propagation to move between the diagnostic points, determined by the phase speed of the detonation wave along the cylinder.

From a common origin at a common time (zero expansion at zero time), the definitive flow direction of the expanding cylinder can be ignored in favour of a simple radial flow consideration. Assuming the phase speed along the cylinder wall is D_0, the time (T) for the cylinder to reach a radial distance r from its initial position, at a position along the cylinder relative to a common point is given by:

$$T(r) = T_{EXP}(r,z) + \frac{z(r)}{D_0} \qquad (1)$$

where T_{EXP} is the experimentally measured (or hydrocode calculated) time at the point in space and z is the distance along the cylinder from a common point.

RESULTS - DETONATION PHASE SPEED

Under plane wave steady conditions, the shock angle and pressure in the copper wall remain unchanged along the length of the cylinder. The phase speed of the wall jump off, determined by the adhered probe array (Figure 1), is therefore also the phase speed of the detonation wave. This would not be true if the wave was changing shape and speed, however by the time the wave reaches the phase speed diagnostics it is believed to have stabilised, hence negating this concern.

Only two of the three phase speed diagnostics were recovered. They showed constant speeds of 8.748 and 8.743mm/μs, with a standard deviation of ±0.1%. All subsequent analysis was carried out with a weighted phase speed of 8.744mm/μs.

RESULTS - EARLY MOTION

Figure 3 shows a typical early motion streak record. The vertical marks are fiducials ground into the base of the glass prism. The record was made on a Cordin rotating mirror camera running at approximately 30mm/μs. By manual digitisation of the trace front, around 1000 data points were recovered from a single trace.

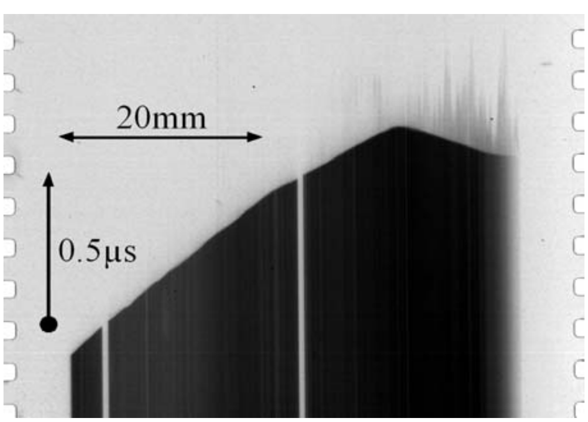

FIGURE 3: EARLY MOTION STREAK TRACE

With the period of revolution of the streak camera known, it is simple to translate this spatial shape into a temporal representation. Simple trigonometry then transforms these distances into the r,z co-ordinates required for the steady analysis described earlier.

Excellent agreement was seen between the early-time expansion histories of the two experiments fielding early motion (Figure 4).

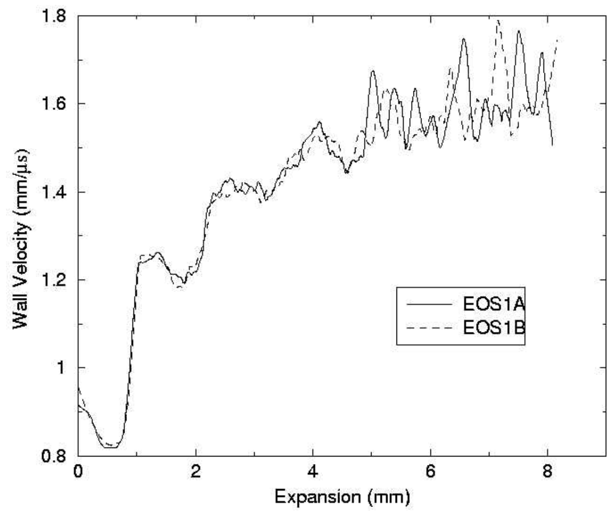

FIGURE 4: EARLY MOTION WALL VELOCITIES

After the initial jump off at about 0.95mm/μs, the wall decelerates because of a rarefaction wave from the free surface, before a reflected shock generates a second velocity jump to 1.25mm/μs. This characteristic behaviour suggests that the copper's tensile stress has not been exceeded; this is important for the EDC37 EoS calculation because no complex fracture models need be included. Subsequent velocity increases are smaller and more spread out as the shock pressure decreases and becomes more of a compression pulse rather than a discontinuity. After 5mm expansion, the deduced velocity is much more erratic, following real perturbations seen on the highly magnified streak record during analysis. This is possibly due to wrinkling or even break-up of the cylinder.

RESULTS - FULL MOTION

To allow analysis of all data boards simultaneously, a Multiple Time Origin (MTO) least squares fitting routine was used to combine all the traces. This proved successful in normalising the data to the optical early motion results. For the third test where no early motion was fielded, the full motion data were normalised to an average of the other two early motion results.

The velocity of the cylinder wall can also be derived from each array measurement, but not in the same detail as obtained from the optical technique. Round-to-round reproducibility is acceptable, although not as good as for the early motion result. Figure 5 shows the entire full motion responses (four per round). The early motion results are superimposed to show the agreement where the two methods overlap.

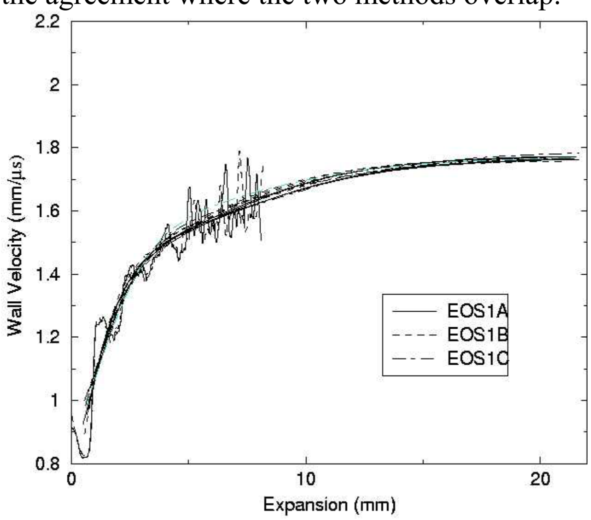

FIGURE 5: FULL MOTION VELOCITIES

Variations in cylinder thickness (and hence mass) and detonation wave symmetry could account for some of the variation. A further source of scatter is the possibility that the probe arrays were not assembled perfectly parallel to the cylinder axis resulting in an increase in the distance of the probe over what was measured, effectively reducing the detonation velocity for the recorded times. However, this would affect the overall expansion and would not explain why the results were less scattered before and after this 5-10mm zone. This region corresponds to the optically erratic part of the early motion optical record, corroborating the theory that the integrity of the cylinder is compromised.

THE REFERENCE FUNCTION

The most sensitive way to compare the hydrocode calculated expansion with that measured from an experiment is by means of time differences. This is done by comparing the code with an arbitrary reference function that is fitted to both the early and full motion results. This means it represents the mean expansion history of the cylinder wall. The function is a linear sum of terms:

$$T_{fit} = a_1 r + a_2\left(1-e^{-0.3r}\right) + a_3\left(1-e^{-0.5r}\right) + a_4\left(1-e^{-0.7r}\right) + a_5\left(1-e^{-r}\right) \quad (2)$$

selected to give a good fit to the overall expansion, but without following the detailed structure revealed by the early motion. Figure 6 shows the experimental time differences from the fitted function representing the mean expansion.

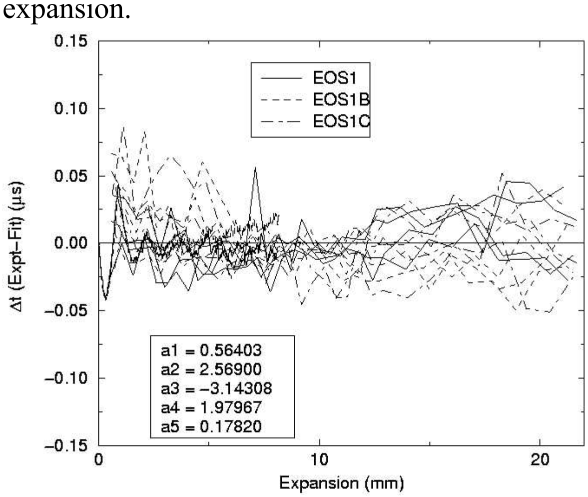

FIGURE 6: REFERENCE FUNCTION ΔT PLOT

With the exception of a few probes in the early stages of the cylinder expansion, this plot shows all of the experimental results to be in a narrow ±40ns band about the fitted function. In particular, the early motion exhibits oscillatory behaviour with respect to the reference function, which will have to be matched by the code when deducing the appropriate EoS parameters for the detonation products. These values are repeatedly changed until the code calculated time differences (from the reference function) match those of the experiment.

SUPPORTING WAVE SHAPE EXPERIMENTS

The first stage in attempting to model the cylinder test experiments in a code is to generate the correct detonation burn times using the WBL model and associated parameters for EDC37. To calculate the correct starting conditions for the propagation code and to validate its output, a short series of supporting experiments were carried out to measure the detonation wave shape at different places along the cylinder. Together with the wave shape at the very end of the cylinder from the third full-size cylinder test, the results of these supplementary rounds will give a robust test of the burn time calculation.

Each experiment was essentially a shorter version of the cylinder test, with the same material and manufacturing constraints, but without the expansion diagnostics. Pin probes were placed in contact with the EDC37 through holes in the metal casing at a number of locations along the length of the experiment. These were of a novel design, using transparent nylon sleeves to insulate the charged probe from the electrically grounded cylinder. These were to further aid confirmation of the burn time calculation by recording detonation wave arrival times. An additional array of phase speed foil probes, as used in the full test, was attached to the outside of the sleeves to increase the timing information available at the early stages of detonation wave travel along the cylinder.

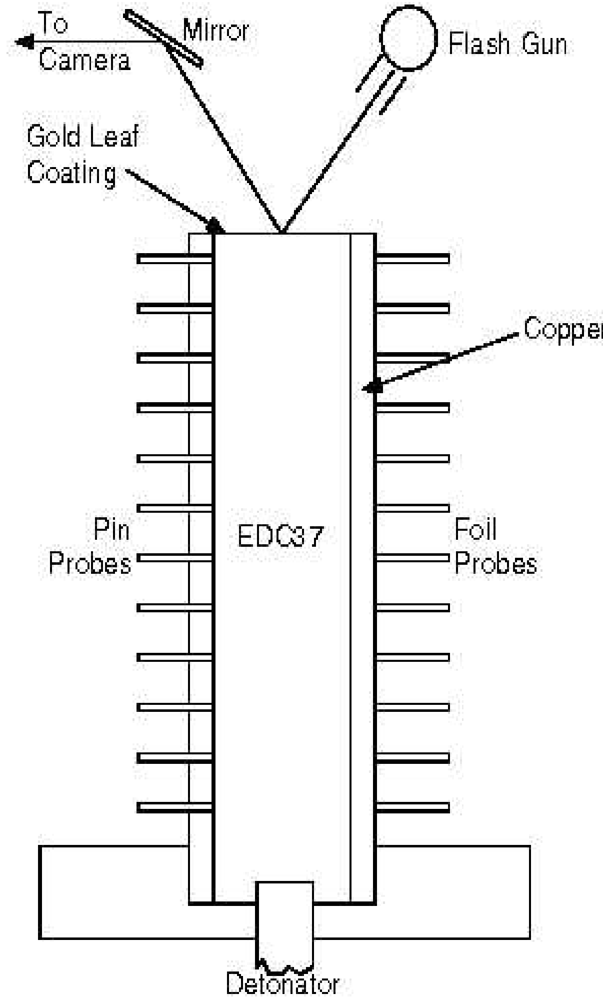

FIGURE 7: WAVE SHAPE EXPERIMENT

The first experiment looked at the wave shape just 12mm from the detonator (using 17mm long charges with a 5mm detonator recess). This was at a distance before the copper could influence the wave and therefore provides information on the effective centre of initiation from the detonator. The second was of length 120mm to confirm the shape of the wave part way along the full detonation distance. As the real cylinder test was really formed from two pieces of explosive with a join, a third experiment was fired which was also 120mm in length, but this time made from two pieces: 90mm and 30mm long. This would show whether having two explosives in series affected the detonation wave shape by comparison with the 120mm single-length trial.

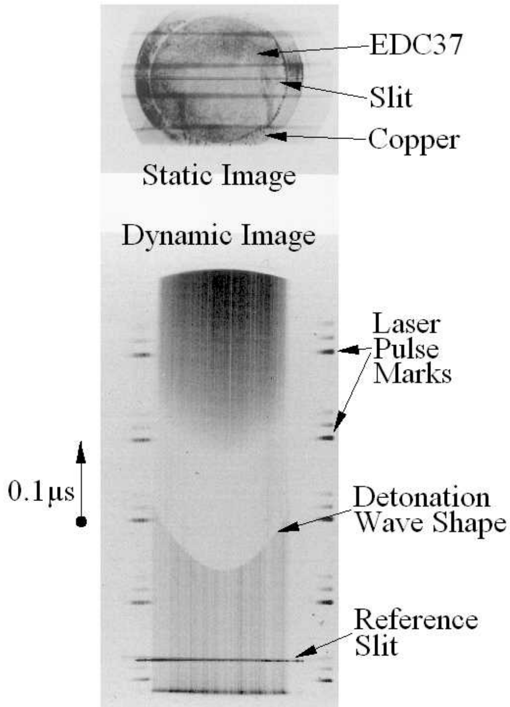

FIGURE 8: 120MM CHARGE STREAK RECORD

The primary diagnostic for these experiments was an optical measurement of the detonation wave shape emerging from the end of the cylinder. A layer of gold leaf 0.1μm thick was placed on the EDC37 to act as a mirror of exactly the same size as the explosive. Light from a conventional camera flash was reflected off this to a streak camera with its slit aligned across the diameter of the charge. The shape of the emerging detonation wave is revealed by extinction of the reflected light from the gold. Light produced from any air gaps, especially any immediately above the gold leaf, is sufficient to swamp the camera's photo-multipliers and hence lose the record. To reduce the possibility of this occurring, silicone fluid was drizzled onto the end of the charge immediately before firing to fill any air gaps that may be present. A typical streak record (including static image taken immediately prior to firing) can be seen as Figure 8. The laser pulses were generated at a known frequency, allowing accurate calculation of the camera writing speed.

For improved temporal resolution, an electronic streak camera was used for these experiments. The limited size of the camera recording window (60mm) meant only a modest increase in writing speed over the maximum obtainable from the rotating mirror camera (35mm/μs cf. 30mm/μs) could be used for the relatively long duration (0.5μs), highly curved wave emerging from the shorter support experiments. The writing rate was increased to around 100mm/μs for the other rounds where the event duration was only about 50ns.

WBL WAVE SHAPE RESULTS

Dynamic magnification was calculated from the width of the record corresponding to the EDC37 diameter, whilst the writing rate was determined from the separation of the laser pulses using a least squares fitting method.

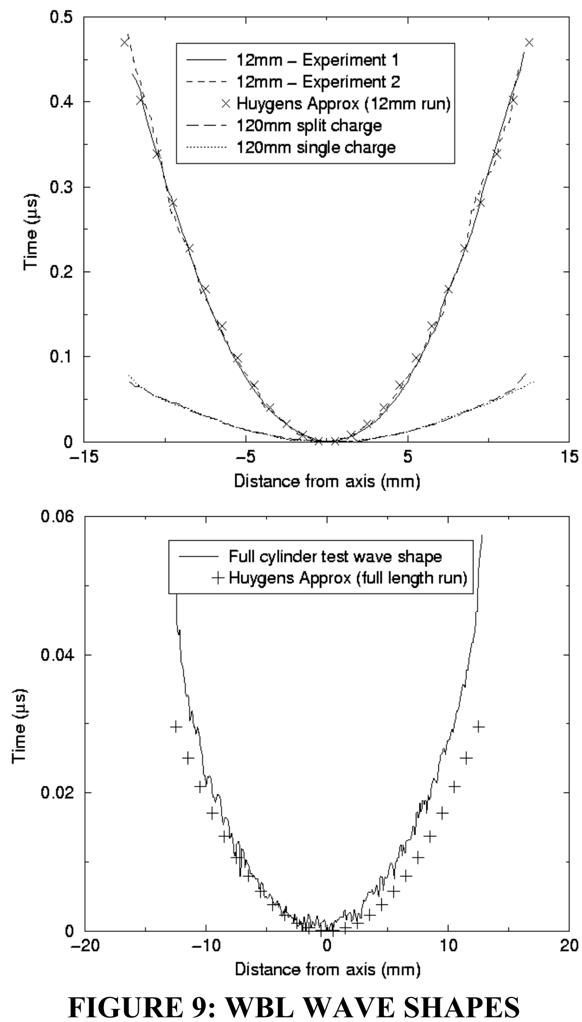

FIGURE 9: WBL WAVE SHAPES

Figure 9 displays the main findings of the supporting experiments. Firstly, the two 12mm long runs did indeed produce Huygens-like behaviour, confirming that a point initiation is a valid method of detonating the cylinder test in the codes. Secondly, the fact that the two 120mm long runs lay almost exactly on top of each other confirms that the presence of a join in the HE does not significantly perturb the wave front for WBL considerations. Finally, the lower graph clearly shows the inadequacy of Huygens approximation over a full cylinder test run distance. With such a variation in the detonation wave angle at the edges of the charge, there are implications for the subsequent motion of the adjacent material. Shock refraction calculations can give an indication of the extent of this effect. With the initial wave front incident on the boundary at an angle near that expected from a Huygens prediction, the shock pressure transmitted into the copper is around 26GPa, resulting in a free surface jump off velocity of 1.0mm/μs (in the direction of movement). However, at the experimentally observed angle, the transmitted shock pressure is nearer 41GPa, corresponding to a free surface velocity of 1.5mm/μs. These are simple calculations with no account taken of attenuation or detonation pressure decrease due to curvature, but they serve to illustrate an effect of not calculating the wave propagation correctly.

WBL WAVE PROPAGATION CALCULATION

When the WBL model was first conceived, the function relating the detonation speed (D) to the local wave front curvature (K) was assumed to be linear:

$$D = D_{CJ}\left(1 - AK\right) \qquad (3)$$

where D_{CJ} is the Chapman-Jouguet speed for a plane wave (K=0.0mm^{-1}), and A is an explosive dependent constant. This form was found to be inadequate for the insensitive high explosive EDC35 and a tabular non-linear relation was created. A similar non-linear relation is also found necessary for EDC37, as seen in Figure 10.

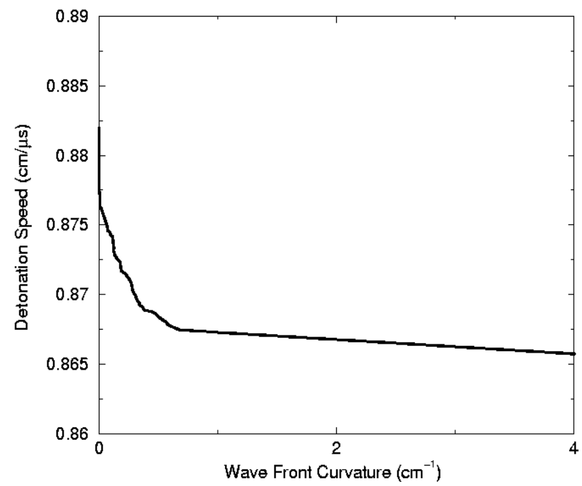

FIGURE 10: EDC37 D(K) RELATION

To run these EDC37 WBL parameters in the cylinder test geometry, calculations were carried out using the 2D arbitrary mesh PSC (Propagation of Surfaces under Curvature) implementation of the WBL equations. In all cases the code was run with a uniform mesh of 0.5mm square cells and a 2.5ns timestep.

An initial wave front has to be defined to start these calculations. A Huygens wave with an apparent centre of initiation 5mm inside the detonator looked reasonable in matching the experiments where boundary effects had yet to occur (Figure 9). Starting with the same conditions, PSC gave a similar fit, but a systematic difference between the calculation and experiment became evident when looking in detail at the residual time differences. Using the sum of square residuals as a guide to the quality of fit, the initiation position was altered until this sum was minimised and the best fit obtained. The optimum initiation pattern was defined by a 1.5mm diameter disc, centred on and perpendicular to the axis, located 2.855mm outside the EDC37.

This input condition was applied directly to the longer geometries of the 120mm charges of the support trials, as well as the copper cylinder test. Figure 11 shows the residual time differences between PSC and the experiments.

There is still some evidence of a systematic variation, but this is less than 10ns and similar to the quality of fit obtained with the original experiments that generated the *D(K)* relation. This agreement demonstrates the ability to correctly calculate the EDC37 detonation wave propagation in the cylinder tests.

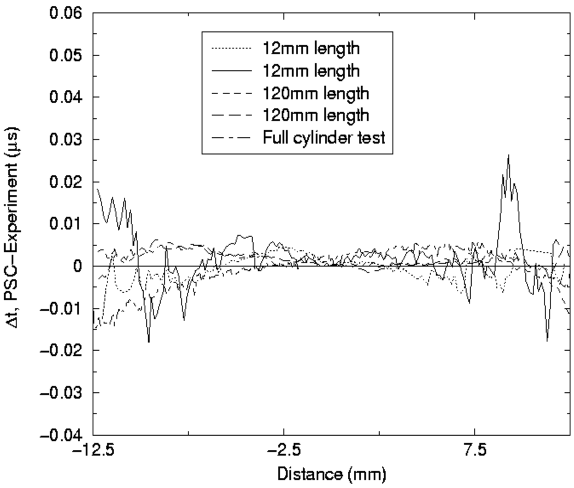

FIGURE 11: PSC CALCULATION RESIDUAL ERRORS

EQUATION OF STATE DETERMINATION

To compare with experiment, accurate tracking of the cylinder free surface and its interception with the diagnostic positions is required from a 2D hydrocode. Firstly, the phase speed of the shock along the cylinder is obtained from linear least squares fitting of data from a diagnostic line, spaced 0.1mm off and parallel to the free surface. This provides the means to generate the radial expansion history from the inclined diagnostic lines using steady analysis. Then two diagnostic lines were used to represent the general positions of the full motion probe arrays, one being 12mm further along the cylinder than the other. Finally a third line represented the inclined prism of the optical early motion measurement.

The Jones-Wilkins-Lee (JWL) EoS for explosive detonation products is

$$P = \frac{\omega E}{V} + A\left(1 - \frac{\omega}{R_1 V}\right) + B\left(1 - \frac{\omega}{R_2 V}\right) \qquad (4)$$

with its isentrope defined by

$$P = A e^{(-R_1 V)} + B e^{(-R_2 V)} + C V^{(-1-\omega)} \qquad (5)$$

where A, B, C, R_1, R_2 and ω are constants, whilst V is the product volume relative to the initial explosive volume, E is the energy per unit volume and P is the pressure. Three simultaneous equations in A, B and C are obtained by applying Equations 4, 5 and its derivative at CJ conditions. These enable a complete set of self consistent parameters to be generated for given values of P_{CJ}, R_1, R_2 and ω which can then be used in the hydrocode calculation.

Changes in the EoS constants have a distinct effect on the pattern of time differences between the calculated and experiment mean expansions. Figure 12 summarises the change in curve shape to be expected by altering the value of each constant. Therefore to obtain the best Δt plot, each variable in the JWL EoS can be varied with prior knowledge of how it will affect the result.

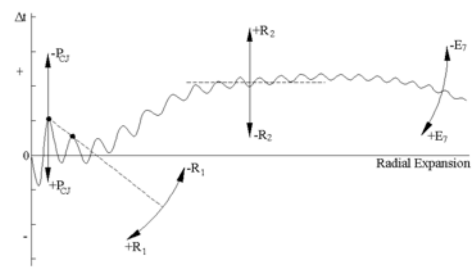

FIGURE 12: EFFECT OF JWL CONSTANTS ON ΔT PLOT

The parameter E_7 referred to in this chart (Figure 12) is the energy corresponding to a 7-fold increase in the volume of the detonation products, the largest measured in the experiments. This is less than the total energy available (E_0) in expanding from CJ conditions to infinity and is readily calculated:

$$E_0 = E_7 + \int_7^{\infty} P\, dV \quad = E_7 + \frac{C\, 7^{-\omega}}{\omega} \qquad (6)$$

since the exponential terms in the adiabat (Equation 5) are negligibly small when V=7.

WBL-CONSISTENT JWL EOS FOR EDC37

With the detonation burn field calculated from PSC, the cylinder test was initially modelled in the hydrocode with a coarse mesh (0.8mm square cells). This mesh size allowed quick investigative runs to be completed. By gradually refining the mesh and altering the JWL parameters in line with Figure 12, the following parameters were found. The time differences between experiment and calculation (via the reference function) can be seen as Figure 13.

TABLE 1: CALCULATED JWL PARAMETERS FOR EDC37

Parameter	Value	Units
P_{CJ}	0.388	Mbar
D_{CJ}	0.8819	cm/μs
V_{CJ}	0.3959906	cc/g
ρ_0	1.841	g/cc
V_0	0.543183	cc/g
E_0	0.0719557	Mbar.cc/g
A	6.6420212	Mbar
R_1	4.25	
B	0.2282927	Mbar
R_2	1.825	
C	0.0188156	Mbar
ω	0.25	

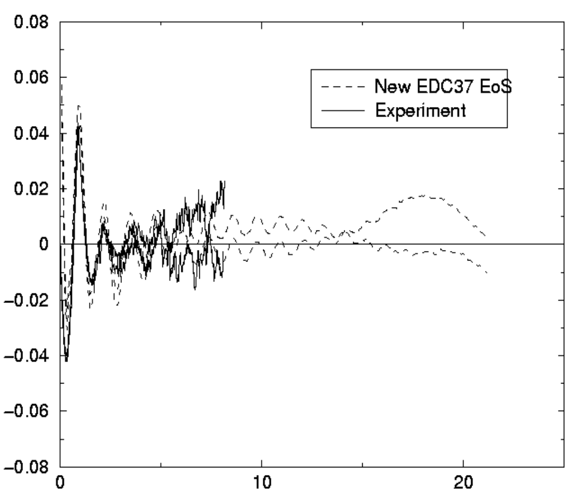

FIGURE 13: NEW JWL EOS CYLINDER TEST ΔT PLOT